omponent of the Hessian

$$) \frac{\partial^2 r_i}{\partial x_k \partial x_j}(\boldsymbol{x})\Bigg).$$

-squares problem is given

$J(x)^T r(x)$.

second derivatives of the ligibly small. In this case, y called the *Gauss-Newton*

$)^T r(x)$.

calculation of the second

9.2, with

$, \ldots, 21$.

rix with elements given by:

$= 1, \ldots, 21$.

vton algorithm to find the $_m, y_m)$. Figure 9.3 shows ss-Newton algorithm. The d $\phi = 0.541$. ∎

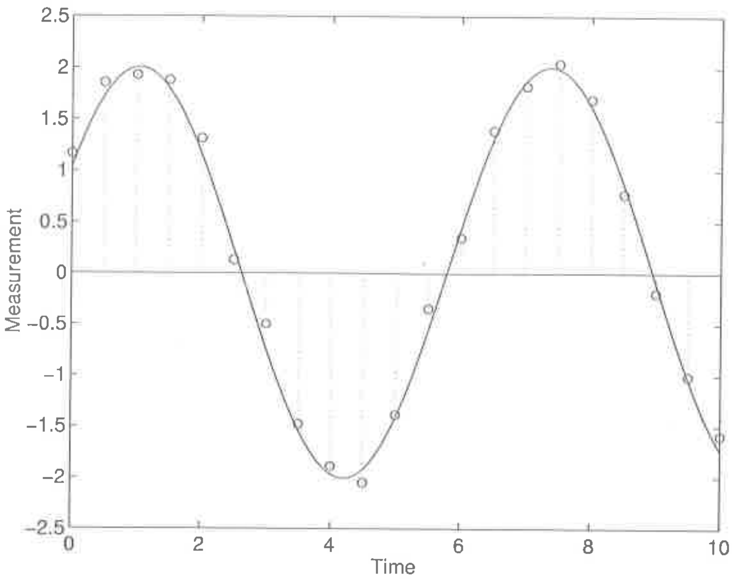

Figure 9.3 Sinusoid of best fit in Example 9.3.

A potential problem with the Gauss-Newton method is that the matrix $\boldsymbol{J}(\boldsymbol{x})^T\boldsymbol{J}(\boldsymbol{x})$ may not be positive definite. As described before, this problem can be overcome using a Levenberg-Marquardt modification:

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} - (\boldsymbol{J}(\boldsymbol{x})^T\boldsymbol{J}(\boldsymbol{x}) + \mu_k \boldsymbol{I})^{-1}\boldsymbol{J}(\boldsymbol{x})^T\boldsymbol{r}(\boldsymbol{x}).$$

The above is referred to in the literature as the *Levenberg-Marquardt algorithm*, because the original Levenberg-Marquardt modification was developed specifically for the nonlinear least-squares problem. An alternative interpretation of the Levenberg-Marquardt algorithm is to view the term $\mu_k \boldsymbol{I}$ as an approximation to $\boldsymbol{S}(\boldsymbol{x})$ in Newton's algorithm.

EXERCISES

9.1 Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (x - x_0)^4$, where $x_0 \in \mathbb{R}$ is a constant. Suppose that we apply Newton's method to the problem of minimizing f.

a. Write down the update equation for Newton's method applied to the problem.

b. Let $y^{(k)} = |x^{(k)} - x_0|$, where $x^{(k)}$ is the kth iterate in Newton's method. Show that the sequence $\{y^{(k)}\}$ satisfies $y^{(k+1)} = \frac{2}{3}y^{(k)}$.

c. Show that $x^{(k)} \to x_0$ for any initial guess $x^{(0)}$.

d. Show that the order of convergence of the sequence $\{x^{(k)}\}$ in part b is 1.

e. Theorem 9.1 states that under certain conditions, the order of convergence of Newton's method is at least 2. Why does that theorem not hold in this particular problem?

9.2 Consider the problem of minimizing $f(x) = x^{\frac{4}{3}} = (\sqrt[3]{x})^4$, $x \in \mathbb{R}$. Note that 0 is the global minimizer of f.

a. Write down the algorithm for Newton's method applied to this problem.

b. Show that as long as the starting point is not 0, the algorithm in part a does not converge to 0 (no matter how close to 0 we start).

9.3 Consider "Rosenbrock's Function": $f(\boldsymbol{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$, where $\boldsymbol{x} = [x_1, x_2]^T$ (known to be a "nasty" function—often used as a benchmark for testing algorithms). This function is also known as the banana function because of the shape of its level sets.

a. Prove that $[1, 1]^T$ is the unique global minimizer of f over $\mathbb{R}^2$.

b. With a starting point of $[0, 0]^T$, apply two iterations of Newton's method. *Hint:*

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

c. Repeat part b using a gradient algorithm with a fixed step size of $\alpha_k = 0.05$ at each iteration.

9.4 Consider the modified Newton's algorithm

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} - \alpha_k \boldsymbol{F}(\boldsymbol{x}^{(k)})^{-1} \boldsymbol{g}^{(k)},$$

where $\alpha_k = \arg\min_{\alpha \ge 0} f(\boldsymbol{x}^{(k)} - \alpha \boldsymbol{F}(\boldsymbol{x}^{(k)})^{-1} \boldsymbol{g}^{(k)})$. Suppose that we apply the algorithm to a quadratic function $f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x} - \boldsymbol{x}^T \boldsymbol{b}$, where $\boldsymbol{Q} = \boldsymbol{Q}^T > 0$. Recall that the standard Newton's method reaches the point $\boldsymbol{x}^*$ such that $\nabla f(\boldsymbol{x}^*) = \boldsymbol{0}$ in just one step starting from any initial point $\boldsymbol{x}^{(0)}$. Does the above modified Newton's algorithm possess the same property? Justify your answer.

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